Design of electric machines using the topology optimization method

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Abstract

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1. Introduction

2. Magnetostatic formulation

Under steady-state conditions (magnetostatics), Maxwell's equations simplify to:

$$\nabla \cdot \vec{B} = 0 \tag{1}$$

$$\nabla \times \vec{H} = \vec{J} \tag{2}$$

Assuming linear and isotropic materials, the constitutive relation is:

$$\vec{B} = \mu \vec{H} \tag{3}$$

By introducing the magnetic vector potential $\vec{A}$ such that $\vec{B} = \nabla \times \vec{A}$ and assuming a 2D formulation with $\vec{A} = A_z(x, y)\,\vec{e}_z$, we obtain:

$$\vec{B} = \left[\frac{\partial A_z}{\partial y},\ -\frac{\partial A_z}{\partial x}\right] \tag{4}$$

Substituting Eq. (4) into Eq. (2), the governing equation becomes:

$$\nabla \cdot \left(\frac{1}{\mu} \nabla A_z \right) = -J_z \tag{5}$$

If μ is constant, this equation reduces to:

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = -\mu J_z \tag{6}$$

This equation is solved numerically using the Finite Element Method (FEM), allowing the magnetic field $\vec{B}$ to be obtained from A_z via Eq. (4). The magnetic force acting on a region (e.g., the armature) is computed using Maxwell's stress tensor:

$$F_{\text{ela,x}} = \frac{1}{\mu_0} \oint_\Gamma \left[\left(B_n^2 - B_t^2 \right) n_x + 2 B_n B_t n_y \right] dl \tag{7}$$

where B_n and B_t are the normal and tangential components of the magnetic field along the boundary Γ of the region of interest.

3. Thermal problem formulation

To model the thermal behavior of a system such as a solenoid carrying electric current, we start from the conservation of energy applied to a control volume.

3.1. Energy balance

Let $\Omega \subset \mathbb{R}^2$ be a domain with boundary $\partial\Omega$. The energy balance for a control volume states:

Energy in - energy out + internal generation = stored energy change

In mathematical form:

$$\frac{\partial}{\partial t} \int_\Omega \rho c_p T \, dV = - \int_{\partial\Omega} \vec{q} \cdot \vec{n} \, dS + \int_\Omega Q_{gen} \, dV \tag{8}$$

where:

- ρ is the density [kg/m^3],
- c_p is the specific heat capacity [J/(kg·K)],

- T is the temperature [K],
- $\vec{q}$ is the heat flux vector [W/m^2],
- Q_{gen} is the volumetric heat generation [W/m^3],
- $\vec{n}$ is the outward normal vector to the boundary.

Assuming steady-state conditions:

$$\frac{\partial T}{\partial t} = 0 \quad \Rightarrow \quad \frac{\partial}{\partial t} \int_\Omega \rho c_p T \, dV = 0 \tag{9}$$

3.2. Fourier's law

Heat conduction is described by Fourier's law:

$$\vec{q} = -\kappa \nabla T \tag{10}$$

Substituting into the energy balance:

$$0 = -\int_{\partial\Omega} (-\kappa \nabla T \cdot \vec{n}) \, dS + \int_\Omega Q_{gen} \, dV \tag{11}$$

Using the divergence theorem:

$$\int_\Omega \nabla \cdot (\kappa \nabla T) \, dV = \int_\Omega Q_{gen} \, dV \tag{12}$$

Since this must hold for any Ω:

$$-\nabla \cdot (\kappa \nabla T) = Q_{gen} \tag{13}$$

3.3. Convective heat exchange

Heat exchange with the surrounding environment (e.g., air around the solenoid) is modeled using a Robin boundary condition. This convective contribution can be treated in two ways:

1. As a boundary condition:

$$-\kappa \nabla T \cdot \vec{n} = h(T - T_{ext}) \quad \text{on } \partial\Omega_h \tag{14}$$

2. Or as a volumetric term (common in simplified 1D/2D models):

$$-\nabla \cdot (\kappa \nabla T) = Q_{gen} + h(T_{ext} - T) \tag{15}$$

This last form is an approximation, used when convection acts uniformly over a surface and can be distributed as an equivalent volumetric term.

3.4. Joule heating from electric current

When heat is generated by electrical current in the solenoid coils, the volumetric source term is:

$$Q_{gen} = \frac{J^2}{\sigma} = \rho J^2 \tag{16}$$

where:

- J is the current density [A/m^2],
- σ is the electrical conductivity [S/m],
- ρ is the electrical resistivity [Ω·m].

3.5. Final governing equation

The final steady-state thermal equation, including Joule heating and convection, is:

$$-\nabla \cdot (\kappa \nabla T) = \rho J^2 + h(T_{ext} - T) \tag{17}$$

4. Design of magnetic actuators

4.1. Material models

Different material models are used depending on the type of optimization (magnetic, thermal, or thermo-magnetic). All models rely on pseudo-density design variables ρ (or p_1, p_2, p_3 for multimaterial cases), which are continuous during optimization and tend toward binary values after projection.

4.1.1. Magnetic optimization (single-material)

In magnetic-only problems, a SIMP model is used to interpolate the relative magnetic permeability of a ferromagnetic material:

$$\mu_r(\rho) = \mu_{\mathrm{air}} + \rho^p(\mu_{\mathrm{ferro}} - \mu_{\mathrm{air}}) \tag{18}$$

No thermal properties are modeled in this case.

4.1.2. Thermal optimization (single-material)

In thermal-only problems, the design variable ρ controls the thermal conductivity κ via a SIMP interpolation model:

$$\kappa(\rho) = \varepsilon + \rho^p(\kappa_{\text{ferro}} - \varepsilon) \tag{19}$$

$$h(\rho) = h_{\text{ar}}(1-\rho) \tag{20}$$

4.1.3. Thermo-magnetic optimization (single-material)

When both magnetic and thermal behaviors are considered simultaneously, the design variable $\rho \in [0,1]$ controls the interpolation of both magnetic and thermal properties using the SIMP model. The following expressions are used:

$$\mu_r(\rho) = \mu_{\text{air}} + \rho^p(\mu_{\text{ferro}} - \mu_{\text{air}}) \tag{21}$$

$$\kappa(\rho) = \varepsilon + \rho^p(\kappa_{\text{ferro}} - \varepsilon) \tag{22}$$

$$h(\rho) = h_{\text{ar}}(1-\rho) \tag{23}$$

4.1.4. Thermo-magnetic multimaterial optimization (discrete material selection)

In this case, three design variables p_1, p_2, and p_3 are used to represent four material options by activating different binary combinations, as shown in the table below:

Discrete material mapping:. Each combination of the design variables (p_1, p_2, p_3) corresponds to a different physical material with its own magnetic and thermal properties, as shown below:

The interpolated properties are defined via the following expressions, constructed to activate the desired material based on the combination of (p_1, p_2, p_3):

Thermal conductivity:.

$$\kappa(p_1,p_2,p_3) = \varepsilon + (200-\varepsilon)p_2(1-p_1)p_3 + (54-\varepsilon)p_1(1-p_2) + (200-\varepsilon)p_1p_2(1-p_3) + (200-\varepsilon)p_1p_2p_3 \tag{24}$$

Convection coefficient:.

$$h(p_1,p_2,p_3) = h_{\text{ar}}\left[(1-p_1)(1-p_2) + (1-p_2)p_1p_3 + (1-p_1)p_2p_3\right] \tag{25}$$

p_1	p_2	p_3	μ_r	κ	h	Material
1	0	1	μ_{ferro}	54	0	Iron
0	1	0	μ_{air}	200	0	Coil
1	1	1	$\mu_{\text{ferro}}/2$	200	0	Composite (XXX)
0	0	1	μ_{air}	0.0264	h_{ar}	Air
0	1	1	μ_{air}	0.0264	h_{ar}	Air

Table 1: Discrete material mapping used in the multimaterial thermo-magnetic optimization.

Relative magnetic permeability:.

$$\mu_r(p_1, p_2, p_3) = \mu_{\text{ar}} + (\mu_{\text{ferro}} - \mu_{\text{ar}})\left[p_1(1-p_2)p_3 + 0.5p_1p_2p_3\right] \tag{26}$$

This approach enables the optimizer to autonomously choose among air, iron, coil, and composite materials, balancing magnetic performance and heat dissipation with high resolution.

4.2. Filtering and projection

To promote manufacturable and physically realistic designs, a density filter and Heaviside projection are employed. The filtered variable $\tilde{p}_i$ is computed as:

$$\tilde{p}i = \frac{\sum j \in N_i w_{ij} p_j}{\sum_{j \in N_i} w_{ij}}, \quad w_{ij} = \max(0, r_{\min} - |x_i - x_j|) \tag{27}$$

The projection is then applied to promote binary solutions:

$$\bar{p}_i = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{p}_i - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1-\eta))} \tag{28}$$

Parameters such as β (projection sharpness) are gradually increased during the optimization.

4.3. Objective function and constraints

4.3.1. Magnetic optimization (armature and core)

The objective function is defined as:

$$\mathcal{J}\text{mag}(\rho) = \frac{1}{2} \int \Omega \mu_r^{-1}(\rho) \left| \nabla A_z \right|^2, d\Omega \tag{29}$$

This formulation encourages the distribution of magnetic material (i.e., higher ρ) in regions where the magnetic vector potential A_z has high gradients, thereby increasing magnetic energy. No thermal effects are considered in this case.

4.3.2. Thermal optimization (core)

The thermal objective is to minimize the total thermal energy in the domain, defined by:

$$U_t = \frac{1}{2} \int_V \kappa(\rho) \nabla T \cdot \nabla T \, dV \tag{30}$$

No magnetic effects are considered in this case.

4.3.3. Thermo-magnetic optimization (core)

In this multi-objective formulation, both magnetic and thermal behaviors are considered simultaneously. The objective function is defined as a weighted sum of the magnetic energy and thermal dissipation terms:

$$\mathcal{J}_{\text{multi}}(\rho) = \omega \, \mathcal{J}_{\text{mag}}(\rho) + (1 - \omega) \, \mathcal{J}_{\text{therm}}(\rho) \tag{31}$$

where $\omega \in [0, 1]$ is a weighting parameter that controls the trade-off between maximizing magnetic performance and minimizing heat dissipation. The design variable ρ interpolates both magnetic permeability and thermal conductivity as in the single-material models.

with $\omega \in [0, 1]$ balancing the magnetic and thermal effects.

4.3.4. Multimaterial thermomagnetic optimization

In the multimaterial case, three design variables (p_1, p_2, p_3) are used to interpolate both magnetic and thermal properties, allowing the optimizer to select among multiple discrete material options (iron, air, coil, composite).

The objective function combines two competing goals: maximizing the magnetic force on the armature and minimizing the thermal energy dissipation. The multi-objective function is defined as:

$$\mathcal{J}(p_1, p_2, p_3) = -\alpha F_{\text{ela,x}} + \beta U_t \tag{32}$$

The optimization is subject to a volume constraint:

$$\frac{V(p_1, p_2, p_3)}{V_0} = f \tag{33}$$

where f is the maximum allowed material fraction.

5. Numerical implementation

The numerical implementation of the topology optimization framework for the solenoid design was carried out using the open-source finite element library FEniCS. The formulation of the problem was encoded in Python using FEniCS's high-level language, which enables efficient simulation of magnetostatic fields and material distribution in complex domains.

The sensitivity analysis required for optimization was performed using the `dolfin-adjoint` library, which provides automatic differentiation capabilities for FEniCS models. This allowed the efficient computation of the gradient of the objective function with respect to the design variables, without manual derivation of the adjoint equations.

The optimization itself was conducted using the TOBS (Topology Optimization of Binary Structures) library, a discrete variable optimizer tailored for problems involving material distribution. TOBS handles the binary nature of topology optimization (material or void) and incorporates filtering and projection techniques to ensure manufacturability and avoid numerical instabilities such as checkerboarding.

A schematic flowchart of the optimization routine is presented in Figure **??**, outlining the iterative process that involves solving the forward magnetostatic problem, computing sensitivities via adjoint analysis, updating the material distribution, and checking convergence.

Convergence was monitored through the relative variation of the objective function between iterations, as well as the stabilization of the material distribution field. The optimization process was terminated when both criteria fell below predefined tolerances.

6. Results and discussion

This section presents the results obtained from the topology optimization of the solenoid under different physical formulations. Each subsection corresponds to a distinct optimization setup.

6.1. Magnetic optimization – core and armature

This case considers only the magnetic behavior of the solenoid. The optimization aims to concentrate magnetic material in regions that maximize the magnetic force on the armature and/or the magnetic energy density.

Two optimized designs are presented: one focused on maximizing magnetic energy (Figure 1) and another focused on maximizing the magnetic force acting on the armature (Figure 2).

Figure 1: Optimized topology for the magnetic energy formulation. High-density regions concentrate around magnetic field gradients.

Figure 2: Optimized topology for the magnetic force formulation. Material is redistributed to maximize the force applied to the armature.

6.2. Thermal optimization – core only

In this scenario, the optimizer seeks to minimize thermal dissipation by rearranging thermally conductive material. The heat is mainly generated by Joule effect in the coil region. The optimized distribution is shown in Figure 3.

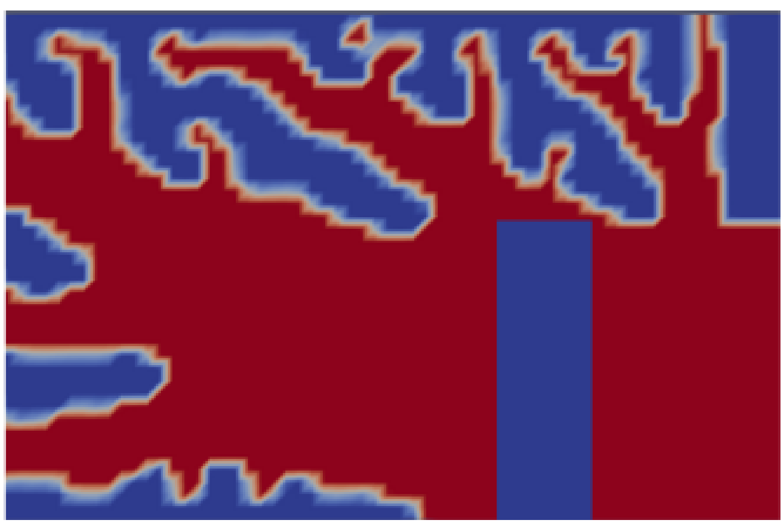

Figure 3: Optimized thermal topology. Thermally conductive material is placed to efficiently dissipate heat from critical regions.

6.3. Thermo-magnetic optimization – single material

In the coupled thermal and magnetic problem, a single design variable is used to interpolate both magnetic permeability and thermal conductivity.

The result, shown in Figure 4, balances thermal and magnetic performance depending on the chosen weighting parameter ω.

Figure 4: Thermo-magnetic optimized topology (single material). The distribution reflects a compromise between thermal dissipation and magnetic force.

6.4. Thermo-magnetic optimization – multimaterial selection

In the final case, the optimizer selects among discrete materials (air, iron, coil, composite) using a multimaterial interpolation scheme. The resulting topology (Figure 5) reflects both physical performance and material allocation efficiency.

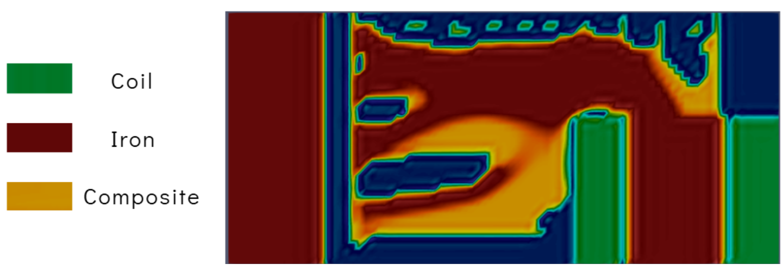

Figure 5: Multimaterial thermo-magnetic optimized topology. The solver autonomously assigns different materials to maximize performance.

7. Conclusions

References